Correlation properties of binary spatiotemporal chaotic sequences and their application to multiple access communication

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This paper studies the correlation property and the spectral density of the binary spatiotemporal chaotic sequences that are generated by the one-way coupled map lattice. The results show that this kind of chaotic sequences possesses excellent randomlike property and could be used directly in the spread spectrum multiple access (SSMA) communications. The multiple access interference in the additional white Gaussian noise background is then analyzed, and the corresponding formulas are presented. The simulation and computation results indicate that the communication system adopting such spreading sequences possesses as good performance as the one employing Gold sequences. But the former has larger capacity and higher complexity. Therefore, the binary spatiotemporal chaotic sequences are good candidates for the spreading sequences in SSMA communications.

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Since the synchronization of chaos was reported in 1990 [1], there has been a great deal of interest in the study of chaotic sequences. Because of the ergodic and randomlike properties [2], chaotic spreading sequences suit the spread spectrum (SS) communications and secure communications quite well [3–6].

Recently, the investigations have been focused on the spatiotemporal chaotic sequences [7–10]. The spatiotemporal chaos systems are a kind of infinite-dimensional dynamic systems. They perform chaotic behaviors in both temporal and spatial directions [11]. One spatiotemporal chaos system could provide large number of chaotic sequences, which couple to each other. So the complexity of these sequences is very high [12]. It is a natural idea to use spatiotemporal chaotic sequences in multiple access (MA) communications [8].

In the earlier papers, the spatiotemporal chaotic sequences generated by the coupled map lattice (CML) model are investigated. The results show that these sequences have good correlation property. This performance is very much expected in SS communications. However, the sequences from a CML model are continuous-state ones. They are not suitable for digital communications. In the practical SS communication system, the spreading sequences must be quantized. But the performance of quantized spatiotemporal chaotic sequences is not studied well.

This paper studies the performance of binary spatiotemporal chaotic sequences. The sequences are generated by the one-way CML (OCML) model [11] and subjected to quantization treatments. The OCML model is described as

$$x_n^0 = x_n,$$

$$x_{n+1}^i = (1-\varepsilon) f(x_n^i) + \varepsilon f(x_n^{i-1}), \quad i = 1,2, \ldots, \tag{1}$$

where n is the discrete time, i is the lattice number. ε is the coupling coefficient and we take $\varepsilon = 0.95$ in the computations. $f(\cdot)$ is simply defined as the Logistic map $f(x) = 4x(1-x)$. x_n is the driving sequence of system (1). Here it is defined as the Chebyshev map [4]

$$x_{n+1} = \cos(k \cos^{-1} x_n), \quad -1 \leqslant x_n \leqslant 1. \tag{2}$$

Quantizing the continuous-state sequences generated in system (1), we get the binary spatiotemporal chaotic sequences

$$y_n^i = \operatorname{sgn}\{x_n^i - E(x_n^i)\}. \tag{3}$$

The correlation property is one of the important factors in measuring the degree of randomness of pseudonoise (PN) sequences. Moreover, in a spread spectrum multiple access (SSMA) system, the correlation property is utilized to improve the performance of the communication system [13]. The spreading sequences with good autocorrelation functions guarantee the least probability of a false synchronization. While the sequences with good cross correlation can help the receiver decrease the influences caused by both noises and other users.

Define the normalized periodic autocorrelation function as

$$\hat{c}_{i,i}(\tau) = \sum_{n=0}^{N-1} y_n^i y_{[n+\tau] mod N}^i, \tag{4}$$

$$c_{i,i}(\tau) = \hat{c}_{i,i}(\tau) / \hat{c}_{i,i}(0), \tag{5}$$

And the normalized periodic cross-correlation function as

$$\hat{c}_{i,j}(\tau) = \sum_{n=0}^{N-1} y_n^i y_{[n+\tau] mod N}^j, \tag{6}$$

$$c_{i,j}(\tau) = \hat{c}_{i,j}(\tau) / \sqrt{\hat{c}_{i,i}(0)\hat{c}_{j,j}(0)}. \tag{7}$$

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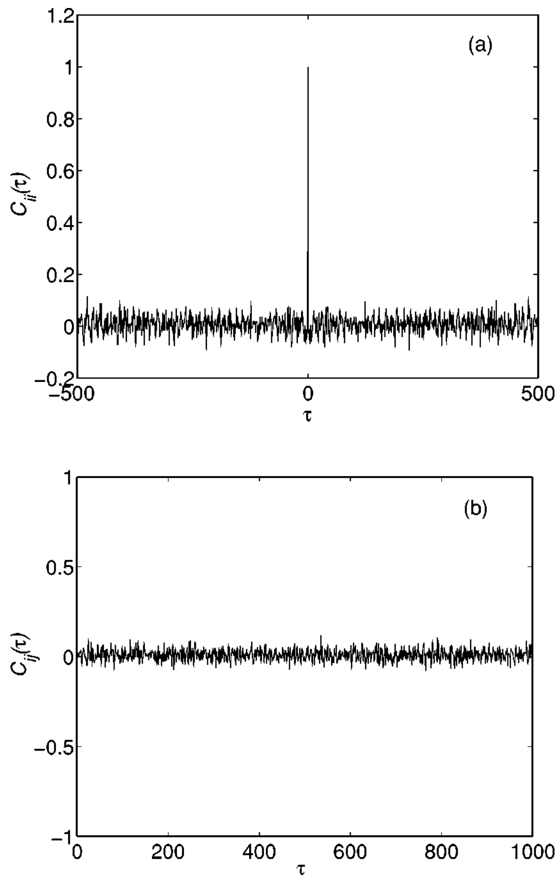

FIG. 1. Correlation properties of binary spatiotemporal chaotic sequences. (a) Normalized periodic autocorrelation function. (b) Normalized periodic cross-correlation function.

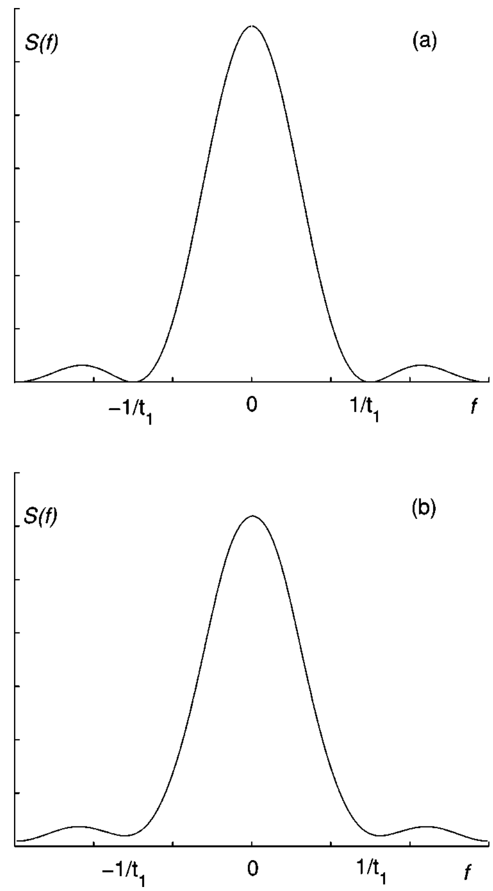

FIG. 2. Spectral density of the binary sequence. (a) Spectral density of the purely random binary sequence. (b) Spectral density of the binary spatiotemporal chaotic sequence.

We could get the correlation functions of binary spatiotemporal chaotic sequences as shown in Fig. 1. The figure indicates that the sidelobe of the autocorrelation function is quite low and the cross-correlation function approaches to zero. Comparing this figure with the correlation functions of continuous-state spatiotemporal chaotic sequences in Ref. [9], we can conclude that, when the length of the sequence is large enough, its correlation property is quite similar to that of the continuous-state one.

The power spectral density is another important measurement of PN sequences. If the binary sequence is considered to be purely random, its spectral density has the form [14]

$$S(f)=t_1\left(\frac{\sin \pi f t_1}{\pi f t_1}\right)^2, \tag{8}$$

and is displayed in Fig. 2(a), in which t_1 is the duration of one bit of the sequence. In fact, the expression has been normalized to represent a signal having unit average power. Unfortunately, the spectral density of binary spatiotemporal chaotic sequence could not be expressed analytically. It could only be expressed numerically. Figure 2(b) gives the numerical result of such spectral density. In fact, the curve is the spectral density average of 100 sequences. The two curves in Fig. 2 are quite similar. This fact illustrates that the degree of randomness of binary spatiotemporal chaotic sequences is quite good.

The analyses of correlation property and the spectral density indicate that the binary spatiotemporal chaotic sequences possess good randomlike property. This feature is very much expected in SS communications.

In the practical MA communication system, many users occupy the same channel. The signal of one user could be considered as interference to the others. For the wireless environment, the noise in the channel is also unneglected. MA interference and the noise are two main factors to influence the performance of SSMA communication system. In the following part, we will evaluate the bit error rate (BER) caused by them.

Set the spreading sequences of uth user and vth user are the binary spatiotemporal chaotic sequences y_n^u and y_n^v, respectively, $n\in\{0,1,\ldots,N-1\}$, N is the length of a spreading sequence. The kth information bit of the uth user is $S_k^u \in\{-1,+1\}$. Its duration is T. After SS modulation, the transmitted signal $S_k^u(t)$ could be written as

$$S_k^u(t)=\sum_{n=0}^{N-1} S_k^u y_n^u g_{T/N}\left(t-(k-1)T-n\frac{T}{N}\right), \tag{9}$$

where $g_{T/N}(t)$ is a rectangular pulse as indicated below

$$g_{T/N}(t)=\begin{cases}1, & t\in[0,T/N],\\ 0 & \text{otherwise.}\end{cases} \tag{10}$$

At the receiver of the uth user, S_k^u can be recovered after correlative demodulation.

$$\Phi_k^{uu}=\frac{1}{2T}\int_{(k-1)T}^{kT} S_k^u(t)Q^u[t-(k-1)T]dt=\frac{1}{2}S_k^u\sum_{n=0}^{N-1}|y_n^u|^2 =\frac{N}{2}S_k^u, \tag{11}$$

where

$$Q^u[t-(k-1)T]=\sum_{n=0}^{N-1} y_n^u g_{T/N}\left(t-n\frac{T}{N}-(k-1)T\right). \tag{12}$$

Assume the signals of the uth user and the vth user are independent in phase as is always the case. The time difference Δt is a zero-mean random variable. So the interference caused by the vth user at the receiver of the uth user can be expressed as

$$\Phi_k^{uv}=\frac{1}{2T}\int_{(k-1)T}^{(k-1)T+\Delta t} S_{k-1}^v(t+T-\Delta t)Q^u[t-(k-1)T]dt +\frac{1}{2T}\int_{(k-1)T+\Delta t}^{kT} S_k^v(t-\Delta t)Q^u[t-(k-1)T]dt. \tag{13}$$

The interference-to-signal ratio cause by the vth user at the receiver of the uth user is

$$I_{u,v}=\frac{\sigma^2(\Phi_k^{uv})}{(\Phi_k^{uu})^2} =\frac{\sum_{m=1-N}^{N-1}[2C_{u,v}(m)C_{u,v}(m)+C_{u,v}(m)C_{u,v}(m+1)]}{6N^3}, \tag{14}$$

in which

$$C_{u,v}(m)=\begin{cases}\sum_{i=0}^{N-m-1} y_i^u y_{i+m}^v, & m=0,1,\ldots,N-1,\\ \sum_{i=-m}^{N-1} y_{i+m}^u y_i^v, & m=-1,-2,\ldots,-N+1,\\ 0 & \text{otherwise,}\end{cases} \tag{15}$$

is the partial cross-correlation function.

Here we assume the strict power control is concerned to equalize the power of each user. Then the average interference of one user to another can be written as

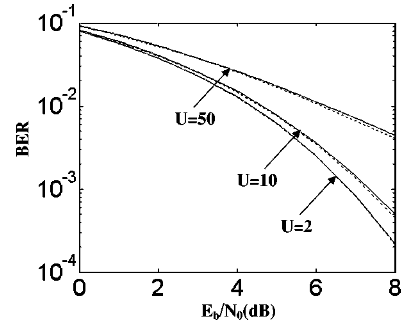

FIG. 3. BER vs E_b/N_0. —, binary spatiotemporal chaotic sequences. ---, Gold sequences.

$$I=E_{u,v}\{I_{u,v}\}, \tag{16}$$

where $E_{u,v}\{\cdot\}$ is the mean value with both u and v varying. Assume that there are totally U users in the system. The signal-to-interference ratio of the entire system can be defined as

$$\frac{E_b}{N_I}=\frac{1}{(U-1)I}, \tag{17}$$

where E_b is the signal energy per information bit and N_I is the interference caused by other users.

Consider the contribution of both MA interference and additional white Gaussian noise (AWGN), BER for a coherently demodulated binary phase shift keying system is [15]

$$\mathrm{BER}=Q\left(\sqrt{\frac{E_b}{N_0/2+N_I}}\right)=Q\left(\sqrt{\frac{1}{\frac{N_0}{2E_b}+\frac{N_I}{E_b}}}\right) =Q\left(\sqrt{\frac{1}{\frac{N_0}{2E_b}+(U-1)I}}\right), \tag{18}$$

where $N_0/2$ is the double-sided noise power spectral density, and $Q(x)=1/\sqrt{2\pi}\int_x^\infty e^{-y^2/2}dy$.

Equation (18) shows the relationship between BER and E_b/N_0 and the user number (U). This relationship is displayed in Fig. 3. In the figure, the results of binary spatiotemporal chaotic sequences are compared with Gold sequences [13]. The latter are well known as a kind of PN sequences generated by modulo-2 addition of a pair of maximal-length linear shift register sequences (m sequences). They possess good correlation property and are widely used in SS communications. In the figure, the length of spreading sequences N is equal to 127. The curves show that the bit-error performances of binary spatiotemporal chaotic sequences and Gold sequences are quite similar. When U is relatively less, their BER curves almost overlap. When U is large enough, the BER performance of binary spatiotemporal chaotic sequences is a little bit worse than that of the Gold sequences. But the difference is nearly negligible.

However, one should note that the binary spatiotemporal chaotic sequences possess some additional advantages beyond Gold sequences. (i) The chaotic sequences exhibit higher complexity. It is reported that the linear complexity of such sequences is $N/2$ [12], where N is the length of the sequence. Gold sequences are generated by modulo-2 addition of a pair of m sequences. So their linear complexity is up to $2\log_2(N+1)$. When N is large enough, the linear complexity of binary chaotic sequence is much higher than the Gold one. (ii) the spatiotemporal chaotic sequence generator can generate much more sequences than the Gold sequence generator does. Therefore, the capacity of the spatiotemporal chaotic SSMA system could be extended to be much larger than the traditional one.

From the investigations in this paper, we can conclude that the binary spatiotemporal chaotic sequences could show the excellent randomlike property. The correlation functions of these sequences are as good as the continuous-state ones. The autocorrelation function is similar to the impulse function and the cross-correlation function approaches to zero. The spectral density of these binary chaotic sequences is similar to the ideal random binary sequences. All these features are expected in SSMA communications. Comparing with the continuous-state chaotic sequences, the binary ones are more practical. The calculation results of BER, considering the MA interference and AWGN channel, show that the MA performance of binary spatiotemporal chaotic sequences is as good as that of the Gold sequences. But the former possesses higher complexity and larger capacity. The theoretical analysis and numerical results conclude that the binary spatiotemporal chaotic sequences are good candidates for the spreading sequences in SSMA communications.

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